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Giving Students Meaningful Work Pages 28-32

Beyond One Right Answer

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Differentiating instruction is a great way to make math meaningful for all. It's just a question of the questions teachers pose.

Consider the following two scenarios. Do they sound familiar?

Scenario 1: A teacher decides that she wants a math lesson to focus on two-digit by two-digit multiplication. She finds an appropriate problem for the students to work on. Although she knows that six or seven students still struggle with the concepts involved in multiplying by even a single-digit number, she presents the problem to all students, making sure that the struggling students receive help from herself or other students.

To her credit, the teacher is attempting to provide support. However, she's not only putting students in a situation that might reinforce their belief that math is just too hard for them, but she's also implying that she doesn't expect them to succeed without help. Is that the message she really wants to deliver?

Scenario 2: A teacher is working on teaching fact families. He asks students to describe the fact family for 3 + 4. One student offers a response: 3 + 4, 4 + 3, 7 - 4, 7 - 3. The teacher records this on the board and checks that other students concur. The whole episode takes less than five minutes, only one student responded, and now the teacher needs to set up another activity.

The teacher is focusing on an important mathematical idea—the relationship between addition and subtraction— but in a fairly narrow way. Students come to view math as a subject in which their job is to quickly answer question after question with the single right answer the teacher expects.

Two Beliefs That Need to Change

Teaching mathematics at the elementary and middle school levels has changed in many ways in the last two decades. Students are more likely to use manipulatives and technology than in the past, teachers are more likely to encourage students to use personal strategies, and there is typically much more discussion in the classroom.

However, two widely held beliefs continue to hold sway in math class—that all students should work on the same problem at the same time (Scenario 1) and that each math question should have a single answer (Scenario 2).

Both scenarios reflect common practice among many hardworking and capable teachers. But what else can teachers do?

An Idea Takes Root

We need to find a way to meet the needs of a broader range of students with richer activities. This approach has multiple benefits: More students experience success with meaningful tasks, more students are engaged, more students see themselves as competent in math, and more students enjoy learning math.

Some background. About 10 years ago, one of my graduate students decided that she was interested in exploring the kinds of questions that math teachers ask during instruction. At the same time, I was researching the various phases of student development in each strand of mathematics.[1] The two universes collided. I started to see the potential for using questioning as a way to differentiate instruction in a classroom with groups of students at different levels.

What emerged was the delineation of two core techniques for differentiating instruction in mathematics in a meaningful but manageable way—using open questions and parallel tasks. Each technique enables students to enter into a mathematical conversation from different access points—one by using a wide net and the other by being deliberately, but thoughtfully, focused.

Open Questions

An open question is just what it sounds like—a single question that is broad enough to meet the needs of a wide range of students while still engaging each one in meaningful mathematics.

Consider, for example, this question: "If someone asked you to name two numbers that are easy to multiply, which numbers would you choose and why?" As long as the word *multiply* is familiar, any student can contribute. Let's look at some student responses:

> *Student 1*: I would choose 2 and 5 because I know that 2 × 5 = 10.
>
> *Student 2:* I would choose 45 × 100 because 45 × 100 means 45 hundreds. That means you write 45 in the hundreds place in a place value chart, so it's actually 4,500.

Thousands	Hundreds	Tens	Ones
	45		

Thousands	Hundreds	Tens	Ones
4	5	0	0

Student 3: I would choose 4 × 9 because you could just double 2 × 9 to get 18 and then double that. Eighteen doubled is two 10s and two 8s, so that's 20 and 16, which is 36.

Student 4: I would choose 4 × 25 because I know 4 quarters make a dollar, and that's 100.

Student 5: I would choose 1 × 34,782 because if you multiply by 1, you don't have to do anything; it's just the other number.

That single question about multiplication ends up reinforcing a wide range of mathematics concepts: place value; working with money; and several properties of multiplication, including multiplying by 1 and the notion that multiplying by 4 is the same as doubling twice.

Open questions also provide choice, one of the elements implicit in differentiating instruction. Students can answer in a way that is suitable for their level. And everyone benefits from different perspectives when they hear other students respond. The open question is both accessible to and enriching for all.

Some teachers may worry that students will not sufficiently challenge themselves—for example, providing simple responses when they are capable of more complex ones. In practice, this happens much less often than one might think; students seem to enjoy challenging themselves when they have the latitude to do so. If, however, a student is consistently taking the easy route, a teacher might privately respond, "Annie, that was a clever answer, but I notice that a lot of people thought of this. Can you think of a more interesting one?" Or, if the student answers more publicly, the teacher might say, "That's a good answer. Can you think of anything else?"

Creating Open Questions

Teachers create open questions by allowing for a certain level of ambiguity.[2] For example, rather than asking for two numbers that add up to 37, a teacher could ask for two numbers that add up to about 40. Or instead of asking for the third angle size in a triangle with one angle of 20° and another of 38°, a teacher could ask for three possible angle sizes in a triangle with at least one narrow (or sharp) angle.

Students may initially be uncomfortable with ambiguity in a subject that has, until now, seemed so clear-cut. However, they almost always warm up to and appreciate the latitude that this ambiguity allows. Four strategies for creating open questions follow.

Strategy 1: Start with the answer. A teacher can take a straightforward question and present it backward. For example, instead of asking, "What is 23 + 38?" a teacher could say, "I added two numbers. The sum is 61. What numbers might I have added?"

Students can provide multiple responses, including 60 + 1, a simple and straightforward answer that might be accessible to students who have struggled with more closed questions.

Teachers can use Strategy 1 to probe student thinking in computation or measurement. They might ask,

- The area of a rectangle is 20 square inches. What might be its length and width?
- A 3D shape has 8 vertices. What might it look like? (Students might suggest a cube.)
- The 10th term in a pattern is 36. What might the 8th and 9th terms be? Describe the pattern. [Students might think 36 = 26 + 10, so the pattern might start 27 (26 + 1), 28 (26 + 2), and so on, with the students realizing that the 8th and 9th terms are 34 (26 + 8) and 35 (26 + 9).]

Strategy 2: Ask for similarities and differences. Asking students how two things are alike and how they are different can provide teachers with valuable assessment for learning information. A teacher might ask,

- How are the numbers 4 and 9 (or 350 and 550 or 100 and 1,000, and so on) alike? How are they different? (Students might point out whether the numbers are even or odd or divisible by numbers other than themselves.)
- How is the formula for the perimeter of a rectangle like the formula for its area? How is it different? (Students might indicate that both formulas involve using values for the length and width of the rectangle, but that one involves addition and the other doesn't.)

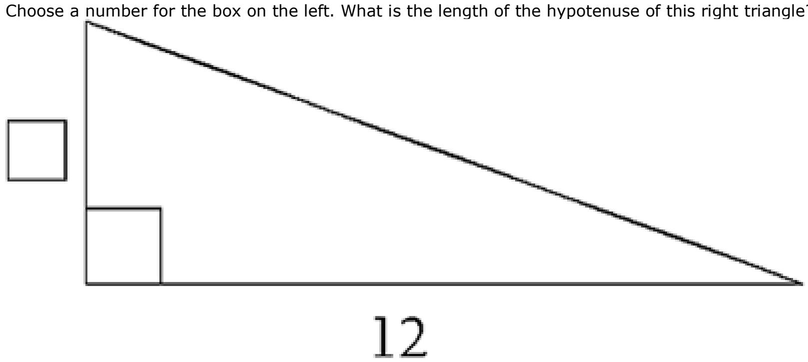
- How are these two patterns alike? How are they different?

  4, 8, 12, 16, 20, …

  4, 7, 10, 13, 16, …

Strategy 3: Allow choice in the data provided. Students are empowered by the opportunity to choose one or more numbers with which to work. For example, teachers might ask,

- Choose a number for the box on the left. What is the length of the hypotenuse of this right triangle?



- Choose a value for the fourth number in the series that follows and calculate the mean: 4, 5, 6, ___.
- A pattern starts at □, and you add △ each time. Choose values for □ and △. Will 40 be in your pattern? Explain.

Strategy 4: Ask students to create a sentence. Asking students to create a sentence using specific mathematics vocabulary is a good way to assess student understanding of the vocabulary and to foster creativity. A teacher might ask,

- Use the words *even*, *more*, and *always*, and the number 10 in a sentence. (Students might say, "If you add 10 to an even number more than 10, the answer is always even and always has at least two digits.")
- Use the words *length, width*, *formula*, and the number 10 in a sentence.
- Use the words *increasing, decreasing, pattern*, and the number 18 in a sentence.

Parallel Tasks

Another approach to differentiation is using parallel tasks. Although test creators think of parallel tasks as having similar levels of difficulty, in this context parallel tasks focus on the same big ideas but have *different* levels of difficulty, thus taking into account the variation in student readiness.

For example, using multiplication to simplify the counting of equal groups is useful no matter the size of the numbers involved in the computation. Parallel tasks might allow students who are ready to deal with only simpler values to use those simpler values, whereas students who are ready for more complex work could use more challenging values. Common questions that focus on strategies and the meaning of multiplication would apply to both tasks.

Creating Parallel Tasks

Just as there are techniques to creating open questions, there are techniques to creating parallel tasks.

Strategy 1: Let students choose between two problems. The teacher might give students a choice between two problems at different levels of difficulty:

- Choice 1: There are 427 students in Tara's school in the morning. Ninety-nine of them left for a field trip. How many students are still in their classrooms? (The problem involves subtraction and is suitable for mental math calculations because 99 is so close to 100.)
- Choice 2: There are 61 students in 3rd grade. Nineteen of them are in the library. How many students are still in the classrooms? (This problem also uses subtraction and is suitable for mental math, but it involves smaller values for students who are not ready for work with 3-digit numbers.)

Strategy 2: Pose common questions for all students to answer. The teacher could ask *all* students the following questions, no matter which task they completed:

- Before you calculated, could you tell whether the number of students left in the classrooms would be more or less than one-half of the total number of students? Explain.
- What operation did you or could you use to solve your problem? Why that one?
- Would it be easier to solve the problem if one more student had left the classroom? Why?
- How could you use mental math to solve your problem?
- How did you solve your problem? How many students are still in their classrooms?

Notice that the questions focus on the common elements—until the last one, in which the teacher asks the students to describe their specific strategies.

Meaningful—and Manageable

Many teachers shy away from differentiation in math because they do not see how to do it. These two strategies—creating open questions and creating parallel tasks—show how to differentiate math instruction in a manageable way. By doing so, teachers can make all students feel like part of the larger community of learners as all contribute to a rich discussion of mathematics.

Open Questions for Every Grade

Grade 1

- The answer is 10. What might the question be?
- How are 5 and 10 alike? How are they different?
- Choose two numbers to add. What is the sum?
- Create a sentence using the words and numbers *and*, *more*, *5*, and *3*.

Grade 4

- The answer is ⅔. What might the question be?
- How are 80 and 800 alike? How are they different?
- Create a sentence using the words and numbers *product*, *8*, *almost*, and *50*.
- The product of two numbers is almost 30. What might the two numbers be?

Grade 8

- The answer is 30π. What might the question be?
- How are the formulas for the circumference and the area of a circle alike? How are they different?
- Create a sentence using the words *surface area, volume, greater*, and *300*.
- The sum of two integers is a negative integer very far from zero. What might the integers be?

Grade 11

- The answer is $\frac{\sqrt{2}}{2}$. What might the question be?
- How are calculating powers and calculating logarithms alike? How are they different?
- Create a sentence using the words *irrational, repeating*, *4*, and *greater*.
- An irrational number is approximately 8. What might it be?

Endnotes

[1] Small, M. (2005). *Prime: Number and operations*. Toronto: Nelson Thomson Learning.

[2] My books *Good Questions: Great Ways to Differentiate Mathematics Instruction* (Teachers College Press, 2009) and *More Good Questions: Great Ways to Differentiate Secondary Mathematics Instruction* (with Amy Lin, Teachers College Press, 2010) provide many models of open questions built around the big ideas in each strand of mathematics for each grade band.

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