Small-group peer discussions are powerful tools for promoting student learning in mathematics (Boaler and Staples 2008; Cohen 1994). Group work succeeds for two reasons: Students must explain mathematics to their peers, and explaining mathematical relationships develops students’ understanding. As a teacher educator, I frequently model the use of small-group discussions in mathematics classrooms, alternating small-group discussions with whole-class discussions and individual work. Many teachers tell me that they enjoy doing math with their peers, but they shy away from using group work in their classrooms because they are worried that English Language Learners (ELLs) will be overwhelmed by the demands of the mathematical discussions. Although this is a commonsense concern, both research and practical experience show that it is possible—and can be very beneficial—to use group work and small-group discussions in linguistically diverse settings.

A good group discussion depends on having a high-quality task to discuss. Measure Twice, which accompanies this article, is one such task (see fig. 1 for teacher notes and the activity sheet for students’ use). I have used the Measure Twice task with middle school students and in teacher workshops. The task focuses on the proportional relationship between ounces and milliliters, and students use real-world objects and a graph to construct the proportional relationship between these two measurements. The task includes follow-up questions in which students explain the relationship by reasoning with multiple representations.

Four major concerns for setting up group work are—

1. selecting appropriate tasks;
2. assigning students to groups;
3. setting up group norms; and
4. assessing students’ understanding.

This article will discuss each concern. First, however, this most critical question needs to be answered: Why use group work in linguistically diverse classes?
Learning to use mathematical language is a major component of learning mathematics (Pimm 1987). Mathematics requires using specialized terms, such as function and set, that take on new meanings. The language of math also includes specific ways of writing and talking. For example, students must learn to define terms and formulate logical arguments. Although numerical calculations can be completed without using words, the actions of mathematical reasoning, modeling, and problem solving require students to use specialized language. With the renewed focus on mathematical reasoning and modeling that is evident in the Common Core State Standards (CCSSI 2010), we will be asking our students to use much more mathematical language in the near future.

Group discussions are one venue in which students can learn to use mathematical language, and this is especially true in linguistically diverse classrooms. When the teacher does most of the talking, students have few opportunities to practice speaking mathematically. Incorporating group work is one way to give students space to learn academic language while absorbing content. Students who might be reluctant to talk in whole-class discussions can practice using mathematical language and get feedback in a relatively low-stakes setting during group work with their peers. Considerations for setting up successful group work in middle school math classrooms, paying particular attention to linguistically diverse classes, are discussed below.

SELECTING TASKS
One of the most critical elements of successful group work is selecting or designing “group worthy” tasks (Cohen 2002). Good group tasks have four characteristics:

1. They focus on important mathematical concepts.
2. They are relatively open-ended and invite collaboration.
3. They have multiple points of entry.
4. They avoid unnecessarily complex language.
These principles of task design apply to all group work situations, not just group work in linguistically diverse classrooms. However, some specific considerations about task design and selection are especially important when students in a class speak many languages.

First, effective tasks focus on important mathematical concepts and allow students opportunities to make connections. Measure Twice incorporates a number of concepts including proportional reasoning and relating multiple representations of a linear relationship. It also highlights the critical connection between the slope of a linear function and the rate of change relating the independent and dependent variables. These concepts and connections are developed as students solve the task, discuss each part, and make generalizations. To maximize the effectiveness of the task, teachers with linguistically diverse classes must plan ahead to ensure that they highlight the key vocabulary and connections as they talk with students.

Second, good group tasks are open-ended problems that require students to think creatively, share ideas, compare solutions, and come to a consensus. The guided but open-ended questions on the Measure Twice activity sheet provoke discussions about the meaning of slope and invite reasoning about why a proportional relationship must contain the origin. Although each question has a correct answer, students will likely use multiple methods to solve questions 6, 7, and 8 and use mathematical language to answer each question. By way of contrast, students are not required to engage in sense-making discussions or use mathematical language to do routine conversion exercises (e.g., 5 oz. = ____ mL).

The third element of high-quality tasks—multiple points of entry—is important for making tasks accessible for a group that speaks many languages. Ideally, a good group task should allow a student with beginning English proficiency to engage in the math at hand. In Measure Twice, for example, a student who is beginning to learn English can engage by gathering data, completing the table of values, plotting points, comparing graphs, and noticing patterns. With some help from a bilingual peer or aide, these students can also develop explanations and make generalizations. In this way, differentiated instruction for a linguistically diverse class does not necessarily require using different tasks for different students but one rich task that can be approached at multiple levels and through multiple modes of participation.

Finally, a good task for linguistically diverse students avoids using unnecessarily complicated language. Gómez, Kurtz, and Jimenez-Silva (2011) provide guidance on reducing the language demands of mathematics word problems. I have used Measure Twice several times, and as Gómez and colleagues recommend, I have refined the wording each time and even changed the page layout to make the task more accessible to all students, but especially to English language learners (ELLs). For example, in an early version of the task, the instructions in step 1 read in this way:

Record the volume of each container in the table below. Put the name of the item in the left column, put the measurement in ounces in the middle column, and the measurement in milliliters in the right-hand column.

I made several changes to make these directions easier to understand:

1. Replaced the word “record” with the equivalent and more familiar word “write.”
2. Added a bulleted list to distinguish each step of instructions.
3. Added a fourth step—writing the
point coordinates in the table. In an earlier version, this was a separate question.

Avoiding unnecessarily complex language does not mean that technical language should be avoided altogether. Group tasks should use and develop academic language in a way that is tied to the instructional goals (and to state standards). For example, in Measure Twice, students must understand and be able to use words like coordinate, axis, and slope. These words are part of the conceptual focus of the task itself, and all students, including ELLs, must learn how to use this vocabulary to meet state and national content standards.

ASSIGNING GROUPS

After choosing an appropriate group task, teachers must select students to work together, making sure to avoid grouping students who do not get along or who get along too well. Beyond these classroom management concerns, three additional questions about group composition must be answered:

1. How many students should be in each group?
2. How should high-, middle-, and low-achieving students be distributed?
3. How should ELLs be assigned to groups?

The ideal group size depends on the task. The group should be large enough to complete the task, but no larger. Working in pairs can be good for written tasks, although tasks that require manipulating multiple tools or making measurements will likely require four students. Most teachers avoid using groups larger than four because there is little opportunity for all students to speak. Groups of four work well because this configuration is flexible. When a task requires pairs of students to work together, students can first work with their elbow partner, before gathering as a group of four to compare answers.

There is general agreement that mixed-level groups are the most beneficial for maximizing students’ learning opportunities. The usual recommendation is to combine some high-, medium-, and low-achieving students in each group (Cohen 2002; Webb and Farivar 1999).

A teacher in a linguistically diverse classroom faces one more issue: the teacher must also consider students’ language backgrounds. There is no single best approach to grouping students by level of English proficiency. Successful teachers take multiple approaches to placing ELL and English-fluent students in groups. For
The Measure Twice Activity and the Common Core State Standards

Grade 7 Standards: Ratios and Proportional Relationships

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.

2. Recognize and represent proportional relationships between quantities.

   a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).

   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and give verbal descriptions of proportional relationships.

   c. Represent proportional relationships by equations. For example, if total cost \( t \) is proportional to the number \( n \) of items purchased at a constant price \( p \), the relationship between the total cost and the number of items can be expressed as \( t = pn \).

   d. Explain what a point \((x, y)\) on the graph of a proportional relationship means in terms of the situation, with special attention to the points \((0, 0)\) and \((1, r)\) where \( r \) is the unit rate. (p. 48)

example, some teachers mix ELLs and non-ELLs from the first day of class; others allow groups of ELLs to work together and do math in their primary language.

For groups of more than two students, many teachers avoid placing one English language learner by himself or herself in a group because that student is likely to be isolated. Some successful teachers tell me that they shift the language composition of their groups over the course of the year. At the start of the year, they assign groups of students who speak the same home language so they can work together. As the year goes on, the teachers mix the groups so that ELLs and their English-dominant peers have more opportunities to work together.

The best balance of students is likely to change from class to class and from group to group. It is critical that a teacher with a linguistically diverse class be aware of the additional layer of complexity that language adds to the mix when assigning students to groups. Being aware of the potential issues that may arise will help him or her be thoughtful about placing students in groups; adjustments can be made throughout the year as what works best for teacher and students becomes apparent.

EStABLISHING GROUP NORMS

Students learn more than just content when they work together; students also learn how to work with their peers. Although group work skills are unlikely to appear on the year-end tests, learning to work with their peers—even those who may not be their friends—is critical. The ability to work well with others is a trait that employers value, so group work prepares students for the working world.

Group projects are also commonly used in college-level courses, especially in business and the laboratory sciences. Therefore, group work can be a very useful tool for equipping students for college and career readiness—a major goal of the Common Core State Standards Initiative (2010).

Teachers play a critical role in helping students learn to work together by setting up norms for positive and productive group interactions. As with group composition, some principles for setting up group norms are the same for all classes, whereas others are specific to linguistically diverse classes.

First, successful group work depends on all students listening to one another, valuing contributions, and responding to ideas (Cohen 2002). This basic norm applies in all classrooms. When it comes to listening, however, issues are specific to a classroom containing those who speak different languages.

In such a classroom, students (and their teacher) need to carefully listen to the mathematical ideas expressed by students who are learning the language of instruction (Moschkovich 1999). At times, this will mean ignoring some grammatical errors in an ELL’s statement to hear what he or she is saying about the math. Middle school students may need extensive encouragement and modeling from their teacher to practice listening to their peers who are not fluent English speakers.

A teacher can promote respectful listening within groups by assigning mathematical authority to students who may not necessarily be considered such by their peers. For example, if students are working on question 6 from Measure Twice, a teacher might listen to a group discussion where one student who is learning English is trying to explain why his or her lines pass through the origin. The teacher can assign authority to this student by building on and revoicing a correct contribution: “I hear Jorge saying that the line showing the relationship between ounces and milliliters passes through zero zero, or \( \text{the origin}. \) What does that tell us about how many milliliters are in zero ounces?”

A second norm specific to linguistically diverse classes is that students’ languages should be valued. Sometimes teachers express concern about students
switching between languages. However, this switching serves many important functions for bilingual learners and can help advance a mathematical discussion (Zahner and Moschkovich 2011). For example, even when instruction is only in English, ELLs can do more mathematical reasoning when they can use all their linguistic resources to make an argument. Therefore, in groups where two or more ELLs speak the same language, the students should be allowed to use their first language to engage in mathematical discussions. A bilingual student can help translate these ideas for the class. This principle shows respect for the resources that ELLs bring to school, and it allows immigrant students to continue learning math as they learn the language of instruction.

ASSESSING UNDERSTANDING
Assessing group work can appear challenging because the teacher must decide whether to assess individual students or the group as a whole. The answer to this conundrum is both. Research has shown that the ideal arrangement is to assess group work using a mixture of group and individual tasks (Cohen 1994). On one hand, if students are only assessed individually, then there is little motivation for them to collaborate during group work time. On the other hand, if the only assessment is a group test, then some students may be tempted to let their group mates do all the work. By using a mixture of group and individual assessment, a teacher can promote a productive collaborative discussion.

One common group assessment is short group presentations in which the teacher asks one group to share its work and asks specific students within the group to share key ideas and show their understanding of the content. For example, after a group works on Measure Twice, a teacher may ask one group to share its graph on the overhead projector. He or she may then call on particular students in the group to explain each question. In classes with linguistically diverse groups, bilingual aides or bilingual students can help students learning English to participate in the group presentation. As noted above, it is important to focus on the mathematical ideas in students’ talk. When an emergent English language learner shares ideas in front of the class, the teacher can model desirable communication.
skills by building on and responding to the student’s mathematical ideas rather than focusing on grammatical correctness.

In addition to being a form of assessment, the group presentation can serve a second important role in linguistically diverse classrooms: As ELLs answer questions, they develop their productive use of academic English. Likewise, as students listen to their peers doing a presentation, ELLs develop their receptive proficiency in English. Since both modes of communication are critical for developing English fluency, this can be an important way to build students’ academic language skills while assessing the quality of a group’s work.

EXPLORING OTHER TASKS

Small-group discussions are one tool that teachers in linguistically diverse classes can use to promote all students’ participation in mathematical discussions. Although many teachers in such classes express reservations about using group work, numerous examples from research and practice have explored teachers successfully using group discussions to teach high-level mathematics to all their students. This article outlined four major elements that a teacher should consider while using groups: selecting tasks, assigning students to groups, setting norms for interactions, and assessing group work.

Measure Twice is one example of a task that a middle school teacher might use for an initial group project. Many more tasks, similar to Measure Twice, are available on NCTM’s Illuminations site (http://illuminations.nctm.org). In addition, NCTM has recently published helpful books about using group work in mathematics classrooms (Horn 2012; Featherstone et al. 2011) and on advancing mathematics education for ELLs (Celedón-Pattichis and Ramirez 2012). The guidelines here are just the start—for the interested reader, the references contain numerous case studies and examples of teachers who have used groups with many different types of students.

REFERENCES


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MEASURE TWICE

Bottles, cans, and other containers have measurements in both ounces (oz.) and milliliters (mL). In this activity, you and your group mates will convert ounces to milliliters using a table, graph, and equation.

Adapted from http://www.flickr.com/photos/roadsidepictures/2530062098/, license C.C. by S.A. 2.0

1. You have 12 containers with measurements in both ounces and milliliters. Each group member should pick three containers with different volumes. See the table below and follow these instructions.

   - Write the names of your items in the left column.
   - Write the volumes in ounces in the second column.
   - Write the volumes in milliliters in the third column.
   - In the fourth column, write the two measurements as a point, \((x, y)\), using ounces for \(x\), and milliliters for \(y\).

The first item, the can of 7-UP®, has already been written in your table.

<table>
<thead>
<tr>
<th>Name</th>
<th>Volume in Ounces (oz.)</th>
<th>Volume in Milliliters (mL)</th>
<th>Coordinates (oz., mL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7-UP</td>
<td>12</td>
<td>355</td>
<td>(12, 355)</td>
</tr>
</tbody>
</table>
2. Plot each point on the graph below. Use the volume in ounces for the x-coordinate and the volume in milliliters for the y-coordinate.

3. Do you see a pattern made by the points? Discuss the pattern with your group mates. Write a sentence to explain what pattern you see. Use as much detail as possible.

4. In your group, stack your graphs on top of one another and line up the axes. Do you see a pattern now? Are any points graphed incorrectly?

5. Use a ruler to draw a line connecting the points in your graph. Does the line miss any points?

6. Where does your line cross the y-axis? Write a sentence to explain why your line crosses the y-axis where it does.

7. What is the slope of the line on your graph? What does the slope tell you about the relationship between ounces and milliliters?

8. Use your graph to make the following estimates.
   a. Estimate the number of milliliters in a juice box containing 7.5 ounces.

   b. Estimate the number of ounces in a 1000 mL (1 liter) bottle of soap.
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